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Exploring Pre-Service Teachers' Algebraic Thinking Levels: A Mixed Method Research

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Abstract

The study was to assess the algebraic thinking levels of prospective basic education teachers using SOLO taxonomy. Sequential explanatory mixed methods design was used for the study. 250 Pre-service mathematics teachers from four selected colleges of education in the Eastern region of Ghana were purposely sampled for the study. The data were collected with the aid of super item test and unstructured interviews as instruments. Descriptive statistics (means and standard deviations) and analysis of variance were used to analyse the quantitative data whereas thematic analysis was employed to analyse the qualitative data. The study revealed that the pre-service teachers showed good algebraic thinking at uni-structural and multi-structural levels of the SOLO taxonomy, but performed abysmally at relational and extended abstract levels. The study recommended that Mathematics tutors at the Colleges of Education in Ghana should use the SOLO model for teaching and evaluation of their students. This would help improve pre-service teacher algebraic thinking levels early enough for preservice teachers' overall progress in teaching mathematics after school.

Keywords: Algebraic thinking levels, Pre-service mathematics teacher, SOLO taxonomy.

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Introduction

Algebra is one of the key curriculum categories offered to help students acquire mathematical knowledge and skills. Research shows that success in algebra is a factor in many other important student outcomes (Knuth et al., 2016). Algebra is deemed tough by Egodawatte and Stoilescu (2015) because it is one of the most abstract threads in mathematics. The use of symbolic language is the hardest aspect of learning algebra for students. Emerging research suggests that students who start an algebra curriculum in the early grades take to the subject better in secondary school (Knuth et al., 2016). As per Ntsohi (2013), algebraic thinking entails employing mathematical symbols and instruments to express information mathematically through various means like verbal descriptions, diagrams, tables, graphs, and equations. It involves the examination of various scenarios, including determining the placement of unknown values, assessing proofs, and seeking evidence.

In a study conducted by Maharaj and Wagh (2016), it was revealed that students should become accustomed to and practice higher-order thinking skills questions among students so that mathematics thinking patterns are no longer something new but must be used to be done in the learning process in the classroom. The goal is to make mathematical thinking patterns a familiar and integral part of the learning process within the classroom. Additionally, it mentions that test instruments, often referred to as questions, are used as measurement tools to assess students' algebraic thinking processes.

Many individuals approach mathematical problem-solving, particularly those involving higher-order thinking skills, by utilizing algebra as a fundamental tool. Algebra holds a central role in the realm of mathematics (Malihatuddarojah & Prahmana, 2019; Ojose, 2011; Star et al., 2015). Algebra is introduced in early education and continues to be a pivotal subject through various educational levels, including analytic geometry, calculus, statistics, trigonometry, and topology (Jupri et al, 2014).

Furthermore, it is highlighted that nearly all activities carried out by humans on a daily basis involve some form of cognitive thought process. This thought process encompasses activities such as conceptualization, problem-solving, decision-making, and the generation of creative ideas (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014). Given the inseparable connection between mathematics and thinking, a prevalent form of thinking when students engage with mathematics is algebraic thinking (Cahyaningtyas, Novita, & Toto, 2018). Algebraic thinking is a critical tool in learning mathematics, particularly in comprehending algebraic concepts and fostering the process of generalizing mathematical ideas based on examples. This process typically involves expressing these generalizations through written or verbal communication (argumentation) that corresponds to the students' age and developmental level (Blanton & Kaput, 2011; Kamol & Har, 2010).

In Ghana, one of the main objectives of mathematics education is to promote higher-order thinking abilities. As a result, evaluating algebraic thinking must go beyond a paper and pencil test that relies solely on recollection and traditional methods and student grades (Owusu-Ansah, Apawu & Akayuure, 2018). There must be a different way altogether of assessing students in Ghanaian classroom in the way they solve mathematics related problems. Curriculum developers need to set some standards in assessing students learning process other than paper and pencil test which we have been doing in our classrooms, Ghana. Biggs and Collis (1982) proposed a cognitive response to problem-solving progresses through different stages, evolving from simple to abstract generally referred to as Structure of the Observed Learning Outcome (SOLO) theory.

Theoretical Framework

Biggs and Collis designed SOLO taxonomy in 1982 as an evidence-based model after researching into samples of learners' thinking in many different areas (Biggs & Collis, 1982). The research found out that learners thinking follow a sequence of increasing structural measure of refinement in many different subjects and across different levels (Hook, 2015). The model was created based on Piaget's stages of cognitive development for the cognitive development of the learners in school learning context.

The SOLO taxonomy model describes new learning outcomes of learners that ranges from simple and robust form into a deep understanding of subjects. The model makes it possible to identify the level or stages at which a learner is currently operating at, and what needs to be done in order to progress in the cause of teaching, learning and assessing a topic.

According to Lim, Wun, and Idris (2010), the SOLO taxonomy was meticulously created to study student responses to assessment of problems in specific areas of mathematics, such as algebra, probability, statistics, geometry, fault, and problem-solving (Biggs & Collis, 1982). In addition to mathematics, the SOLO taxonomy has been approved for use in a number of other fields of study, including poetry, history, geography, science, economics, and evaluating attitudes about teen pregnancies (Collis & Davis, 1986; Biggs & Collis, 1982), as cited in Hatties & Brown (2004). The SOLO taxonomy as an assessment tool assesses learners' knowledge and skills in answering questions under five primary levels: the pre-structural level, uni-structural level, multi-structural level, relational level, and extended abstract level.

Pre-structural Level

This is a level of incompetence where the learner has no knowledge of the task or the subject. The pupil merely receives disconnected information at this point that is neither organized or made sense. The learner is still unable to comprehend the material; hence he fails to show understanding. This kind of response shows a lack of ability to meaningfully respond to the question. Such a response can entail focusing on some unrelated information that has nothing to do with the concept being assessed. Reaching this level in a student assessment denotes incompetence.

Uni-structural Level

This stage refers to a learner's initial level of understanding where they have limited knowledge about a particular task or subject. At this stage, the student has a basic grasp of the fundamental concept but lacks a deeper understanding of its broader implications. They can make simple and straightforward connections, but they are not yet aware of the broader context or significance of the information. In their response, students demonstrate a clear understanding of the task, but their focus is primarily on one specific aspect of it.

Multi-structural Level

At this stage, the student possesses a diverse array of individual data pieces, but they are not integrated into a cohesive whole. The learner has accumulated a broad range of relevant, standalone knowledge. While the learners may recognize connections between various elements, they still struggle to fully comprehend how these components fit together to form a coherent picture. Concepts and ideas related to a particular topic remain disjointed and unconnected. Although the students can identify some connections, they are unable to grasp the significance of the overall context. Responses from the students are evaluated independently, even though they may draw upon relevant factors.

Relational Level

At this level, learners can integrate ideas or facts into a whole, recognize relationships and connect ideas to each other. They understand relationships between theory and practice, purposes and significance of ideas. Learners' answers to questions provide explanations that relate relevant details, which often bring concrete facts together. Verbs such as explain causes, compare and contrast, analyze, relate, distinguish, etc. Children utilizing an algorithm at this level would be able to look for mistakes and inconsistencies as well as recreate missing algorithmic components, according to a UNICEF (United Nations Children's Fund) report (UNICEF, 2007).

Extended Abstract

At this stage of learning, students have the capacity to synthesize information by combining various ideas or facts into a cohesive whole. They possess the ability to identify relationships between concepts and establish connections among different ideas. They also grasp the connections between theoretical knowledge and its practical application, comprehending the purposes and significance of various concepts. When answering questions, learners at this level offer comprehensive explanations that incorporate pertinent details, often consolidating concrete facts. They employ action verbs such as "explain causes," "compare and contrast," "analyze," "relate," and "distinguish" to demonstrate their higher-order thinking skills.

The Current Study

The worldwide learning catastrophe is actually a teaching catastrophe (Oketch, Rolleston & Rossiter, 2021). This suggests that the quality of teachers play a pivotal role in enhancing students' learning outcomes. This assertion is supported by the persistent underperformance of students in Basic Education in Ghana, which has raised concerns about the effectiveness of instruction within Ghanaian classrooms, as noted by Buabeng, Ntow, and Otami (2020). In essence, the statement underscores the critical link between teacher quality and student performance in education, particularly in the context of Ghana (Buabeng, Ntow & Otami, 2020). Furthermore, Owusu-Ansah et al (2018) argue that students' algebraic thinking processes, which are essential for their long-term use of mathematics in decision-making and problem-solving, have been neglected or receives insufficient attention. Prior to 2018, teachers in Colleges of Education in Ghana used to teach and assess Mathematics Content and Methodology as separate entities. However, with the introduction of the Transforming Teacher Education and Learning (T-TEL) program, tutors are mandated to teach Mathematics Content and Methodology together. Additionally, the curriculum emphasis on both conceptual and procedural knowledge in the teaching and learning of mathematics (Salifu, 2021). For students to effectively grasp algebra, it's crucial that their teachers possess a profound understanding of the subject. Teacher training programs should offer the necessary content knowledge to help future educators comprehend algebra to make it easier for prospective teachers to teach the subject upon graduation (Brown & Bergman, 2013). The study seeks to assessed the algebraic thinking levels of pre-service teachers in CoE using SOLO taxonomy.

According to Lian, Yew, and Idris (2010), as thinking levels progress, especially from lower to intermediate relational and upper relational to extended abstract, the problem with students' thinking abilities becomes more pronounced. In a similar study conducted by İncikabi and Biber (2016) concerning the challenges prospective elementary mathematics teachers face in understanding the concept of functions, it was revealed that their understanding of functions in mathematics can improve with effective teaching methods. Additionally, most of their knowledge levels were categorized as pre-structural, multi-structural, and relational, with only a few reaching the extended abstract level (İncikabi & Biber, 2016). In this present study, among the four main categories of the SOLO taxonomy, Pre-service teachers for basic schools are mainly at the uni-structural and multi-structural levels.

Method

The study employed sequential explanatory mixed methods design. The method was convenient in the study because it offset the weakness of both quantitative and qualitative research methods. This is due to the fact that, in a situation where the quantitative method cannot be used to interpret context in which people behave, the qualitative method was used. The quantitative method makes it easy to generalize findings to the large group. From the accessible population, a sample size was selected. From this accessible population, a sample size of 250 was also selected to represent the population. The sample size was selected from 4 main colleges of education in the region including College 1, 2, 3 and 4. This sample is deemed large enough to represent the population and be used to generalize on the study population. It is also selected based on the researchers' ability to collect data on these participants within the stipulated time period and financial constraints available to them.

The study employed simple random sampling to select the four Colleges of Education in Eastern region and used purposive sampling to select the individuals within the four Colleges to respond to the super-item test question with respect to SOLO taxonomy, prospective mathematics teachers in basic education were selected. In undertaking the simple random sampling, the researchers' assigned unique identified by giving each of the colleges a unique identifier; that is the colleges were named "college 1, college 2, college 3, etc." Using a randomization method, the researcher selected four colleges from the population whilst ensuring that each school has an equal chance of being selected. The availability and willingness of the selected schools to participate in the study were verified before administering the instrument towards the data collection.

Data Collection Instruments

The study used cognitive test also known as Super Item test and an unstructured interview as instruments for the data collection. The test consisted of two (2) questions designed based on SOLO taxonomy super-item test format. The unstructured interview guide consisted of items which demanded prospective basic education teachers to explain their working process during the Super Item test.

Results

The results of the study were in two folds. The first part deals with the algebraic thinking levels of prospective mathematics teachers which were assessed using descriptive statistics through quantitative data analysis method. The second part uses qualitative approach to explore the perceptions of students about the reasons behind their inabilities to attain the requisite algebraic levels based on the SOLO taxonomy.

Quantitative Findings

The algebraic thinking levels were aligned with the SOLO taxonomy. As a result, there is at least a question assessing each of the various levels of the SOLO taxonomy except that the information contained at the pre-structural level is not used in extensive decision making since it is assumed that the participants already possess that knowledge even before their enrolment into the school. Details on the SOLO taxonomy and the algebraic thinking levels of participants is shown in Figure 1.

The Figure 1 presents the number of correct and wrong responses, as well as the frequencies and percentages for each level and method/answer combination. It provides insights into the distribution of understanding across different cognitive levels and highlights the performance of specific methods or answers.

It is observed from Figure 1 that in the pre-structural level, uni-structural level and the multi-structural levels only the answers to the questions were needed for the study since these levels are not so much concerned with the processes an individual goes through in arriving at the answer. This is the reason for the absence on the methods in those levels as shown in Figure 1. However, the processes or methods employed in arriving at the answers are crucial in the relational and extended abstract level.

As shown in Figure 1 the SOLO taxonomy is divided into five levels of algebraic thinking: Pre-structural, Uni-structural, Multi-structural, Relational, and Extended abstract. Each level corresponds to a different stage of cognitive development and indicates the depth of understanding and problem-solving ability.

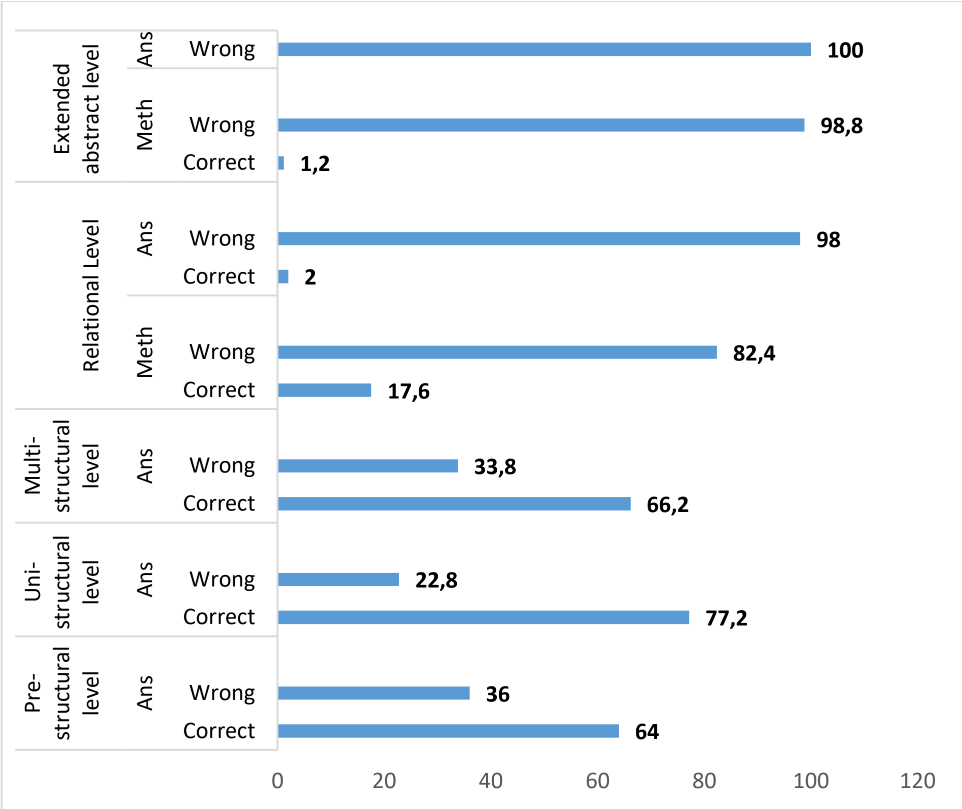

Figure 1. Algebraic Thinking Levels of Preservice Basic School Teachers.

Source: Survey Data (2023)

Qualitative Findings

In an interview with some of the students to ascertain the student's perceptions about the reasons behind their inabilities to attain the requisite algebraic levels based on the SOLO taxonomy, the following are some of the responses;

Researchers: Why do you think you got Question 2A wrong?

Student A: Since the first arrangement has 2 match boxes at the base and the 2nd arrangement also has 3 match boxes at the base, I decided to count the res based on that pattern I saw from the match boxes. I can see that in each arrangement, the base increases by one. But I don't know how come I could not do it well and I had the answers wrong.

It could be deduced from this student A's response that s/he understood the question, was able to identify a pattern but could not algebraically deduced any rule to arrive at subsequent arrangements

Students B: I saw a pattern but was doubting the answer. I did not get 6 match boxes. I was actually thinking that what I was doing were wrong because I was afraid of deviating. I was nervous so I can't tell what exactly I was doing there. I knew I was going to get it wrong.

Student B was able to correctly identify that there was a pattern but could not relax and follow the pattern to arrive at any answer. S/he was probably engulfed by fear and anxiety.

Researchers: How did you get the answers correct?

Student C: Sir, this question is more or less like sequence and series, so I quickly remembered. The 1st arrangement of the match boxes is 3 with 2 boxes at the base, the 2nd arrangement is 6, the base boxes were increased by 1 to make 3 and then the 3 boxes from the 1st arrangement was added to it making 6. I then realized that the 3rd arrangement is 10 with the 4 boxes at the base and 4 plus 6 which was the total for the 2nd arrangement making 10. this gave me the clue and so, I got 21 because it's observed that the base is increased by one as it is moving to the next arrangement of match boxes in shelf and the total number in the previous arrangement is added to it. But sir I could not generate a formula for it.

Discussion

The study intense to explore pre-service teachers algebraic thinking levels which employed sequential explanatory mixed methods design. The study used cognitive test also known as Super Item test and an unstructured interview as instruments for the data collection. The test items were designed based on SOLO taxonomy super-item test format. The unstructured interview guide consisted of items which demanded prospective basic education teachers to explain their working process during the Super Item test.

The Pre-structural level represents the lowest level of understanding. At this level, only 36% had their answers wrong which means that 64% of the participants had their pre-structural level responses correct. Those who failed to answer the algebra questions correctly at this level tend to have issues of comprehension indicating that though they were able to read the questions correctly, they were unable to understand the requirement of the question and process it accordingly. This is an indication that some of the participants (36%) cannot calculate the exact duration for each of the period they were asked at this level. There are several other factors that could prompt students' inability to perform well at the pre-structural level. For instance, during the interaction with the students after marking the test, some of the students stated that lack of fundamental knowledge in algebra, misconceptions about the topic, difficulty with application, limited vocabulary, confusion and sometimes frustration might be some of the causes of students' inability to perform basic operation in solving algebra questions. Similarly, in Kenya, Mbugua et al. (2012) identify similar factors contributing to low mathematics achievement. In the Philippines, Peteros et al. (2020) discovered that a majority of students (53.01%) scored below the average in mathematics in 2020, indicating that many teachers encounter difficulties in making the subject enjoyable for their students.

At the uni-structural level, participants were expected to observe a match box and produce a pattern from it. Students' responses showed that preservice teachers show some understanding of algebraic concepts. Most of the students performed well by scoring 77.2% of the question at this level correctly whilst 22.8% wrongly answered the questions indicating that most of the participants were able to develop the expected pattern. Although students understanding was better as compared to the pre-structural level, preservice teachers were primarily focusing on solving individual aspects of algebraic problems.

Participants were expected to use the same principle of developing a pattern from the arrangement of the match box to solve higher order arrangements. This was used to assess the multi-structural level of the SOLO taxonomy. At this level, the prospective teachers demonstrated more comprehensive understanding of algebraic thinking.

Most of the preservice teachers performed well at this level by obtaining 66.2% of the answers correct. This result indicates that the students were able to use the knowledge on the uni-structural level and extend it to solve higher related patterns. Participants at this stage were expected to integrate multiple concepts and apply them to solve algebraic problems. In a similar study conducted by Ozdemir & Goktepe Yildiz (2015), they discovered that pre-service elementary mathematics achievement test pertaining to spatial orientation skills were typically on a multi-structural level based on SOLO taxonomy. It means that pre-service mathematics teachers were able to extend their understanding to the next level based on SOLO taxonomy.

The relational level shows a deeper understanding of the relationships and connections between different algebraic concepts. Here, participants were asked to write a mathematical rule for the shop keeper to use when arranging the match box. Participants response showed that 17.6% of the prospective teachers used the correct method but only 2% of those had their answers correct. This means that 82.4% of the prospective teachers used wrong methodological approach in solving relational level question. Generally, 98% of the preservice teachers had wrong solution to relational level question. These scores suggest that the prospective teachers struggle to write the mathematical rule from the given task, apply relational thinking and have difficulties in explaining the connections between different algebraic ideas.

The extended abstract level represents the highest level of understanding and indicates the ability to think critically, generalize, and transfer knowledge to new contexts. At this level, participants were asked to develop a mathematical equation to generalize the arrangement. The solutions presented by the preservice teachers showed that almost all the prospective teachers could not solve the question under the extended abstract level of algebraic thinking. The results revealed that only 1.2% of the prospective teachers had their methods correct with 98.8% having wrong methods to solve the questions. Interestingly, even the 1.2% who had their methods correct could not get their answers right. This means that no individual was able to produce the expected equation at this level. These scores suggested lack of ability on the part of preservice teachers to think at an extended abstract level and apply algebraic thinking to complex and abstract situations. The results of this study were in line with the findings of Lian Yew, and Idris (2011), who observed that a majority of pre-service secondary mathematics teachers performed at the lower relational level (focused on algebraic thinking), while a minority performed at higher levels. According to Lian et al. (2011), as thinking levels progress, especially from lower to intermediate relational and upper relational to extended abstract, the problem with students' thinking abilities becomes more pronounced.

In general, the result showed that the prospective mathematics teachers have varying levels of understanding and proficiency in algebraic thinking. Nonetheless, the participants were unable to extend their understanding levels to solve complex and critical situations. Prospective mathematics teachers therefore lack the deep understanding and creativity needed to progress through all the levels of the SOLO taxonomy. The finding suggests that participants have a solid understanding of the basic concepts and can make connections between various aspects of algebraic thinking. However, the preservice mathematics teachers struggle when it comes to applying their knowledge to new and complex situations or generalizing their understanding. In other words, they have a good grasp of the foundational knowledge and can work with the information in a structured manner, but face challenges in transferring that knowledge to novel or real-life contexts. Therefore, participants learning is limited to specific instances and lack the ability to go beyond the immediate scope.

This implies that prospective mathematics teachers understanding of algebraic concepts is limited to basic knowledge and procedural skills. At the pre-structural level, the students may have difficulty grasping the fundamental concepts of algebraic thinking, resulting in unrelated or incorrect answers. However, as they progress to the uni-structural level, they demonstrated understanding of one aspect or component of algebraic thinking. This means they were able to perform basic calculations and identify patterns in simple algebraic expressions. At the multi-structural level, the students also demonstrated understanding of multiple related aspects or components of algebraic thinking and they were able to solve simple equations, recognize patterns in different contexts, and perform calculations. However, their understanding was still fragmented, and they struggled to integrate their knowledge and apply their knowledge to more complex problems.

Further, it was realised that in terms of the relational and extended abstract levels, the students algebraic thinking was poor. At the relational level, it was expected that preservice mathematics teachers could make connections and understand the relationships between various concepts and procedures in algebra. They were also expected to solve more complex equations, analyze and manipulate algebraic expressions, and apply algebraic thinking in a variety of contexts. However, the preservice teachers struggled to solve the questions which required abstract thinking, and make connections between different algebraic concepts. This was reflected in the extended abstract level as they could not exhibit a high level of thinking and abstraction. The preservice teachers were expected to be able to transfer their knowledge and apply it creatively to new and unfamiliar situations. But they struggled to generalize their understanding and apply algebraic thinking beyond familiar contexts.

The results of this study align with the findings of Lian et al. (2011), who observed that a majority of pre-service secondary mathematics teachers performed at the lower relational level (focused on algebraic thinking), while a minority performed at higher levels. According to Lian et al. (2011), as thinking levels progress, especially from lower to intermediate relational and upper relational to extended abstract, the problem with students' thinking abilities becomes more pronounced.

In a similar study conducted by İncikabi and Biber (2016) concerning the challenges prospective elementary mathematics teachers face in understanding the concept of functions, it was revealed that their understanding of functions in mathematics can improve with effective teaching methods. Additionally, most of their knowledge levels were categorized as pre-structural, multi-structural, and relational, with only a few reaching the extended abstract level (İncikabi & Biber, 2016). Meanwhile, Putri, Mardiyana, and Saputro (2017) suggest that students with moderate self-efficacy can progress through the thinking levels of the SOLO taxonomy, starting from uni-structural and multi-structural stages, whereas those with low self-efficacy tend to remain at the pre-structural and uni-structural levels.

Conclusion

The study found that, among the four main categories of the SOLO taxonomy, Pre-service teachers for basic schools are mainly at the uni-structural and multi-structural levels. The study therefore, concluded that the Prospective mathematics teachers have better understanding and application of algebraic concepts at the pre-structural, uni-structural and multi-structural levels whereas their algebraic thinking ability at the relational and the extended abstract levels were low.

Recommendations

The study recommended that educators in the Colleges of Education who teach mathematics should incorporate the SOLO model teaching as well as an alternative assessment tool to gain valuable insights into their students' early problem-solving abilities, allowing them to monitor the overall growth of their students' problem-solving skills. To identify and address students' challenges effectively, math tutors should also be attentive to their students' cognitive levels. The utilization of the SOLO taxonomy can assist educators in staying mindful of their students' cognitive abilities in algebra.

Limitation of the Study

In this present study, it is important to note that only the eastern region out of Ghana's sixteen regions was chosen as the study's location. Additionally, out of the 46 public colleges of education in Ghana, only four were included in the study. While the strategic selection of these colleges aimed to attract students from various parts of the country, it hindered the ability to generalize the findings to the entire nation. Moreover, the study had time constraints, which restricted the range of activities or tasks that prospective mathematics teachers could be assessed on. To address these limitations, future investigations with larger participant groups and a broader array of activities and assessment tools may produce different outcomes.

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Author(s)’ Statements on Ethics and Conflict of Interest

***Ethics Statement*:** We hereby declare that research/publication ethics and citing principles have been considered in all the stages of the study. We take full responsibility for the content of the paper in case of dispute.

***Statement of Interest*:** We have no conflict of interest to declare.

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